THE LOW ENERGY EFFECTIVE THEORY AND NUCLEON STABILITY

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We show that the Standard Model Lagrangian, including small neutrino masses, has an anomaly-free discrete Z_6 symmetry. Anomaly cancellation requires the number of family to be 3 *mod* 6. This symmetry can ensure the stability of the nucleon even when the threshold of new physics Λ is low as 10^2 GeV. All $\Delta B = 1$ and $\Delta B = 2$ (B is the baryon number) effective operators are forbidden by the Z_6 symmetry. $\Delta B = 3$ operators are allowed, but they arise only at dimension 15. We suggest a simple mechanism for realizing reasonable neutrino masses and mixings even with such a low scale for Λ.

The Standard Model (SM) has been highly successful in explaining all experimental observations in the energy regime up to a few hundred GeV. However, it is believed to be an effective field theory valid only up to a cutoff scale Λ. Non-renormalizable operators which are gauge invariant but suppressed by appropriate inverse powers of Λ should then be considered in the low energy effective theory. The dimension 5 operator $\ell\ell HH/\Lambda_L$ (ℓ is the lepton doublet) which violates lepton number (L) by two units is the lowest dimensional of such operators. Experimental evidence for neutrino masses suggests the effective scale of L-violation is around $\Lambda_L \sim 10^{14} - 10^{15}$ GeV. The $d = 6$ operator $QQQ\ell/\Lambda_B^2$ violates both baryon number (B) and lepton number and leads to the decay of the nucleon. The current limits on proton lifetime are $\tau_p > 5 \times 10^{33}$ yrs for $p \to e^+\pi^0$ [1]. These limits imply that $\Lambda_B > 10^{15}$ GeV. Grand Unified Theories with or without supersymmetry generate such B-violating operator with $\Lambda_B \sim 10^{15} - 10^{16}$ GeV. These theories are currently being tested through nucleon decay. Any new physics with a threshold Λ less than the GUT scale will thus be constrained by both proton lifetime and neutrino masses.

As we know the SM effective lagrangian does not have a continuous anomaly-free symmetry that can suppress baryon number and lepton number violating processes. This is our reason for focusing on discrete sym-

metries. It is preferable that such symmetries have a gauge origin[2] since all global symmetries are expected to be violated by the quantum gravitational effects. Discrete gauge symmetries have been utilized in suppressing nucleon decay[3] as well as in addressing other aspects of physics such as solving the μ problem[4] of supersymmetry, fermion mass hierarchy problem[5] and the stability of the axion[4,6]. A Z_3 baryon parity was found in Ref. [3] that suppresses nucleon decay. In order for it to have a gauge origin, complicated particle content were introduced.

We pointed out the SM lagrangian has a discrete Z_6 gauge symmetry which forbids all $\Delta B = 1$ and $\Delta B = 2$ baryon violating effective operators. This can be seen as follows. The SM Yukawa couplings incorporating the seesaw mechanism to generate small neutrino masses is

$$L_{\rm Y} = Qu^cH + Qd^cH^* + \ell e^cH^* + \ell\nu^cH + M_R\nu^c\nu^c \ . \tag{1}$$

Here we have used the standard (lefthanded) notation for the fermion fields and have not displayed the Yukawa couplings or the generation indices. This lagrangian respects a Z_6 discrete symmetry with the charge assignment as shown in Table 1. From Table 1 it is easy to calculate the Z_6 crossed

	Q	u^c	d^c	ℓ	e^c	ν^c	H
Z_6	6	5	1	2	5	3	1

Table 1. Family-independent Z_6 charge assignment of the SM fermions and the Higgs boson.

anomaly coefficients with the SM gauge groups. We find the $SU(3)_C$ and $SU(2)_L$ anomalies to be: $A_{[SU(3)_C]^2\times Z_6} = 3N_g$ and $A_{[SU(2)_L]^2\times Z_6} = N_g$ where N_g is the number of generations. The condition for a Z_N discrete group to be anomaly-free is: $A_i = \frac{N}{2} \ \ mod\ N$ where i stands for $SU(3)_C$ and $SU(2)_L$. For Z_6, this condition reduces to $A_i = 3\ mod\ 6$, so when $N_g = 3$, Z_6 is anomaly-free. The significance of this result is that unknown quantum gravitational effects will respect this Z_6. It is this feature that we utilize to stabilize the nucleon. Absence of anomalies also suggests that the Z_6 may have a simple gauge origin.

We have found[7] a simple and economic embedding of Z_6 into a $U(1)$ gauge symmetry associated with $I_R^3 + L_i + L_j - 2L_k$. Here L_i is the ith family lepton number and $i \neq j \neq k$. No new particles are needed to cancel gauge anomalies. With the inclusion of righthanded neutrinos $I_R^3 = Y - (B-L)/2$

is an anomaly-free symmetry. $L_i + L_j - 2L_k$, which corresponds to the λ_8 generator acting in the leptonic $SU(3)$ family space, is also anomaly-free. The charges of the SM particles under this $U(1)$ are: $Q_i = (0,0,0)$, $\quad {u_i}^c = (-1,-1,-1)$, $\quad {d_i}^c = (1,1,1)$, $\quad l_i = (-4,2,2)$, $\quad {e_i}^c = (5,-1,-1)$, $\quad {\nu_i}^c = (3,-3,-3)$, $\quad H = 1$. This charge assignment allows all quark masses and mixings as well as charged lepton masses. When the $U(1)$ symmetry breaks spontaneously down to Z_6 by the vacuum expectation value of a SM singlet scalar field ϕ with a charge of 6, realistic neutrino masses and mixings are also induced[7].

From Table 1 it is easy to see that the Z_6 discrete symmetry allowed only $\Delta B = 3$ effective operators with lowest-dimension $d = 15$ and forbids all $\Delta B = 1$ and $\Delta B = 2$ operators. $\Delta B = 3$ and $d = 15$ operator will lead to "triple nucleon decay" processes where three nucleons in a heavy nucleus undergo collective decays. We choose a specific operator $Q^5 {\bar{d}^c}{}^4 \bar{\ell}/\Lambda^{11}$ as an example to study the process $pnn \to e^+ + \pi^0$ triple nucleon decay process. In this case the triple nucleon decay lifetime can then be estimated to be

$$\tau \sim \frac{16\pi {f_\pi}^2 \Lambda^{22} R^6}{P^2 \beta^6 M_{{}^3H}}, \tag{2}$$

where $\beta \simeq 0.014$ GeV^3 is the matrix element to convert three quarks into a nucleon [8], $f_\pi = 139$ MeV is the pion decay constant, P is the probability for three nucleons in Oxygen nucleus to overlap in a range the size of Tritium nucleus, R is the ratio between the radii of Tritium nucleus and Oxygen nucleus. By putting the current limit on proton lifetime of 3×10^{33} yrs, we obtain: $\Lambda \sim 10^2$ GeV. Thus we see the Z_6 symmetry ensures the stability of the nucleon.

If the threshold of new physics is low as a few TeV, neutrino mass induced through the effective operator $\ell\ell HH/\Lambda$ will be too large. We found a mechanism by which such operators can be suppressed by making use of a discrete Z_N symmetry (with N odd) surviving to low scale.

Consider the following effective operators in the low energy lagrangian:

$$L \supset \ell\ell HH \frac{S^6}{\Lambda^7} + \frac{S^{2N}}{\Lambda^{2N-4}} \ . \tag{3}$$

Here S is a singlet field which has charge $(1,\ 3)$ under $Z_N \times Z_6$ while ℓ has charge $(-3,\ 2)$. (The Z_6 charges of SM particles are as listed in Table 1.) In this case, if $\Lambda = 10$ TeV and $S = 10^2$ GeV, the neutrino mass is of order $O(0.1)$ eV, which is consistent with the mass scale suggested by the atmospheric neutrino oscillation data.

Two explicit examples of the Z_N symmetry with $N = 5$ and 7 are shown in Table 2. These Z_N symmetries are free from gauge anomalies. In the Z_5 example, the crossed anomaly coefficients for $SU(3)_C$ and $SU(2)_L$ are $5N_g$ and $5N_g/2$ respectively showing that Z_5 is indeed anomaly-free. For Z_7, these coefficients are $7N_g$ and $7N_g/2$, so it is also anomaly-free.

Field	Q	u^c	d^c	ℓ	e^c	H	S
Z_5	1	4	4	2	3	0	1
Z_7	1	6	6	4	3	0	1

Table 2. Z_N charge assignment for $N = 5$ and 7.

It is interesting to ask if the Z_N can be embedded into a gauged $U(1)$ symmetry. A simple possibility we have found is to embed this Z_N into the anomalous $U(1)_A$ symmetry of string origin with the anomalies cancelled by the Green-Schwarz mechanism [9]. Consider $U(1)_{B-L}$ without the right handed neutrinos but with the inclusion of vector-like fermions which have the quantum numbers of $\mathbf{5}(3)$ and $\bar{\mathbf{5}}(2)$ under $SU(5)\times U(1)_A$. This $U(1)_A$ is anomaly-free by virtue of the Green-Schwarz mechanism. When this $U(1)_A$ breaks down to Z_5, the extra particles get heavy mass and are removed from the low energy theory which is the $Z_6 \times Z_5$ model.

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